Tensile performance of prestressed steel elements

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Abstract

Prestressed steel trusses can offer efficient structural solutions for long span applications such as aircraft hangars, stadia and warehouses. In this study, the tensile behaviour of prestressed steel elements, which comprise tubular steel members with internal prestressing cables, is investigated. The stability of the elements under prestress and the load–deformation response of the prestressed elements to the subsequent application of tensile loading are examined analytically, numerically and experimentally, with good correlation achieved between the three approaches. The benefits of prestressing, in terms of increased member strength and stiffness, are demonstrated, and optimal prestress levels are investigated.

1. Introduction

For long-span structural systems, where self-weight becomes an increasingly dominant component of the design loading, significant material savings can be achieved through the use of high tensile steel cables in conjunction with conventional steelwork. Additional benefits can be gained by prestressing the cables, inducing internal forces in the structure that can counteract the applied external loads and control self-weight deflections.

Cable-in-tube systems, whereby the steel cables are housed within hollow structural sections, offer a practical means of realizing the aforementioned concept, and bring further advantages such as ensuring the stability of the tube under the prestressing forces due to the stabilizing action of the internal tensioned cable. Examples of recent applications of prestressed cable-in-tube systems include the reconfiguration of the Sydney Olympic stadium and the Five Star Aviation hangar at Brisbane Airport, both in Australia [1].

Early work on the prestressing of steel structures was described by Magnel [2] in 1950, where it was shown experimentally that improved economy can be achieved by prestressing truss girders. More recent studies have explored the behaviour and design of prestressed steel beams [3, 4], columns [5, 6, 7, 8, 9], trusses [10, 11] and space trusses [12, 13]. Studies of the structural response of sub-assemblies and the overall response of prestressed frames with sliding joints have also been carried out [14, 15, 16, 17], as has a numerical investigation into the stress-erection process of such systems [18]. Each of the above described studies identified potential economies and enhanced performance through the use of prestressing.

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In the system currently being investigated, developed by the company S$^2$ Space Solutions [19], the prestressed cables, housed within the bottom chord of the tubular arched trusses, apply a compressive force to the chord members, which is opposite in nature to the resultant forces arising from the externally applied gravity loads. The behaviour of trussed elements in the above described scenario, i.e. pre-compressed chord elements subsequently subjected to external tensile loading, is the subject of investigation in the present paper. The case of wind uplift, which would result in additional compressive loading of the prestressed element, will be examined in a companion paper.

Analytical and numerical models that predict the tensile behaviour of the cable-in-tube system, including the axial response during prestressing and under the application of tensile loading, and the stability of the system under prestressing are developed in Section 2. Experimental studies are described in Section 3, while comparisons between the test and analytical results and discussions thereof, are made in Section 4.

2. Analytical modelling and numerical verification

In this section, the key behavioural aspects of prestressed steel elements are described, including their response to prestressing and under the subsequent application of tensile loading. The stability of prestressed elements during the prestressing stage, the optimal level of prestress and tensile capacity are also investigated.

2.1. Axial response under prestress

During the prestressing stage of cable-in-tube systems, a tensile force, $P$, is applied to the cable and a compression force of equal magnitude is induced in the tube, as depicted in Figure 1, where $L_0$ is the original length of the cable and tube, $x_c$ is the extension of the cable and $x_t$ is the shortening of the tube.

![Figure 1: (a) Force diagram of cable during prestress; (b) force diagram of tube during prestress; (c) tube and cable locked in position after prestress.](image)

When the prestressing process is complete the cable and the tube are locked in position at both ends such that there is no relative movement between the two components, as shown in
Figure 1(c). Since the prestressing force, \( P \), of the same magnitude, but opposite in direction, is applied to the cable and the tube, the prestressed element is self-equilibrated when the tube and cable are attached at both ends. Therefore, provided no out of plane displacements (i.e. no buckling) are induced during prestressing the equilibrium condition for the prestressed system can be stated as in Equation (1).

\[
[P]_{\text{cable}} - [P]_{\text{tube}} = K_c x_c - K_t x_t = 0, \quad (1)
\]

where \( K_c \) and \( K_t \) are the axial stiffnesses of the cable and the tube respectively. The respective stiffnesses are given by the following expressions:

\[
K_c = \frac{E_c A_c}{L_o}, \quad K_t = \frac{E_t A_t}{L_o}, \quad (2)
\]

where \( E_c \) and \( E_t \) are the Young’s moduli of the cable and the tube respectively with \( A_c \) and \( A_t \) being the cross-sectional areas of the cable and the tube respectively.

2.2. Stability under prestress

Previous analytical studies [20, 21] have shown prestressing elements (i.e. the cables, in the context of the present study) can stabilise their compressed counterparts (i.e. the steel tubes housing the cables) provided that there are sufficient connection points between the two elements. The connection points have been shown to offer effective lateral restraint and thus reduce the buckling length of the compressed element. If the cable is in constant contact with the tube, cable-in-tube systems have infinite connection points, which implies an infinite elastic buckling load. Buckling under prestress would not therefore be anticipated in these systems.

To verify the above analytical findings, a finite element model was implemented using the commercial finite element (FE) analysis software, ABAQUS [22]. The tube to cable cross-sectional area ratio \( (A_t/A_c) \) and Young’s modulus ratio \( (E_t/E_c) \) were 8.96 and 1.58, respectively. Prestress was introduced through thermal loading and the coefficient of linear expansion of the cable, \( \alpha_c \), was set as \( 12 \times 10^{-6} K^{-1} \).

In the numerical simulation, truss elements were used to model the cables since they can only resist axial forces while beam elements, which possess both axial and flexural stiffness, were used to model the tube. The cable and the tube were attached using tie constraints at the ends and constraint equations were applied at various connection points along the length of the tube and cable to prevent relative out of plane motion between the two components.

The number of connection points was varied between one (at mid-length) and three (at quarter points). Finite intervals were chosen since in the experimental investigation of Section 3, the cable was significantly smaller in diameter than the steel tube and connections between the cable and the tube were made by means of collars spaced regularly along the member length. This is described in Section 3.

The numerical modelling was conducted in two stages. In the first stage, an elastic buckling analysis of the tubular element under the prestressing action of the cable was carried out and the buckling mode shapes were obtained. In the next stage, a geometrically nonlinear analysis utilizing the Riks arc-length solution technique [23], was performed with the first eigenmode
from the elastic buckling analysis as an initial imperfection with an amplitude of $L_0/100000$. Incremental prestress was applied to the cable through thermal loading. As illustrated in Figure 2, the FE results are in close agreement with the analytical results and show that the buckling load $N_{cr}$ with respect to that of the unsupported member $N_E$, increases in proportion to $1/L_{cr}^2$, where $L_{cr}$ is the spacing between the connection points (or collars).

![Figure 2: Buckling response of tubular elements under the action of prestressing forces induced through cables connected to the tubular elements at regular intervals along the length. (a) Analytical and numerical results for buckling behaviour of prestressed element. (b) Buckled configuration of tubular element with one connection point to prestressing cable.](image)

2.3. **Behaviour of prestressed system under tension**

Analytical expressions for the axial load-displacement equilibrium paths of prestressed elements under tensile loading are derived in this section. Three cases are considered in the following sub-sections: Case I refers to the scenario where the tube yields prior to the cable; Case II refers to where the cable yielding prior to the tube; and Case III considers the two elements yield simultaneously. The case which actually arises depends on the geometric and material properties of the tube and cable, and the level of prestress, as described in the following sub-sections.

2.3.1. **Case I: Tube yields prior to cable**

The three stages of behaviour that occur for Case I are illustrated in Figure 3, where the adopted notation is defined. The ratios of geometric and material properties between the tube and cable, as well as the level of applied prestress determine whether the tube yields before the cable or vice versa. The tube yields prior to the cable if:

$$\frac{f_{cy}}{E_c} - \frac{P}{A_cE_c} > \frac{f_{ty}}{E_t} + \frac{P}{A_tE_t}, \quad (3)$$
where $f_{cy}$ and $f_{ty}$ are the yield stresses of the cable and the tube respectively.

**Stage 1: Onset of loading until tube yielding**  
$[0 \leq x_1 \leq x_{ty} + x_t]$

The prestressed element, with the tube initially in compression and the cable in tension, both of magnitude $P$, is subjected to a tensile load $N$ as shown in Figure 3.

![Diagram](image)

**Figure 3:** Generalised coordinate definitions for Case I.

The initial compressive axial force in the tube unloads until a tensile displacement of $x_t$ is obtained, from which point the tube is in net tension. The cable, on the other hand, has an initial tensile axial force and it will continue to store tensile strain energy, under increasing $N$. In the sign convention adopted, positive values denote tension and elongation and negative values denote compression and shortening, with the axial displacement $x_1$ measured from the state of the system at the end of the prestressing process. Stage 1 applies from the onset of loading until the tube yields in tension at $x_1 = (x_{ty} + x_t)$, as seen in Figure 3(a). During this stage, both the tube and the cable remain elastic and the total potential energy function $V$ for the system can be expressed thus:

$$V = \frac{1}{2}K_c(x_1 + x_c)^2 + \frac{1}{2}K_t(x_1 - x_t)^2 - Nx_1.$$  

An expression for the equilibrium path for this loading stage, Equation (5), is then obtained by differentiating the total potential energy function $V$ with respect to the axial generalized coordinate, $x_1$, and satisfying the equilibrium condition by setting the resulting expression to zero:

$$N = (K_c + K_t)x_1.$$  

The forces in the tube $N_t$ and cable $N_c$ can therefore be expressed as:

$$N_t = K_t(x_1 - x_t),$$  

$$N_c = K_c(x_1 + x_c).$$
Stage 2: From tube yielding to cable yielding \([0 \leq x_2 \leq (x_{cy} - x_c) - (x_{ty} + x_t)]\)

In the second stage, as shown in Figure 3(b), the tube has yielded while the cable continues to store elastic tensile strain energy as before. This stage continues until the cable yields at an axial displacement relative to the state of the system at the end of prestressing \(x_1 = (x_{cy} - x_c)\) or, relative to the end of Stage 1, at an axial displacement \(x_2 = (x_{cy} - x_c) - (x_{ty} + x_t)\). The potential energy \(V\) now becomes:

\[
V = \frac{1}{2}K_c [x_c + (x_{ty} + x_t) + x_2]^2 + \frac{1}{2}K_t x^2_{ty} + K_{ty}x_2 + \frac{1}{2}K_{st}x^2_2 - Nx_2
\]

and the applied load \(N\) with the component forces \(N_t\) and \(N_c\) are given thus:

\[
N = K_c[x_c + (x_{ty} + x_t)] + K_{ty}x_2 + (K_c + K_{st})x_2, \\
N_t = K_{ty} + K_{st}x_2, \\
N_c = K_c[x_c + (x_{ty} + x_t) + x_2],
\]

where \(K_{st}\) is the stiffness of the tube after yielding allowing, where appropriate, for strain hardening.

Stage 3: From cable yielding to fracture \([0 \leq x_3 \leq x_t - (x_{cy} - x_c)]\)

Stage 3 is the final stage of loading, in which both the tube and the cable have now yielded, and are potentially strain hardening, as shown in Figure 3(c). The axial displacement \(x_3\) is measured from the position at which the cable yields (i.e. the end of Stage 2). Stage 3 continues until either fracture of the tube or, more likely, fracture the cable at a displacement relative to the state of the system at the end of prestressing of:

\[
x_t = L_o \left( \epsilon_{cf} - \frac{P}{A_c E_c} \right) \text{ or } L_o \left( \epsilon_{tf} + \frac{P}{A_t E_t} \right),
\]

where \(\epsilon_{cf}\) and \(\epsilon_{tf}\) are the fracture strain of the cable and tube respectively. Following similar procedures to the previous loading stage, the expression for \(V\) is:

\[
V = \frac{1}{2}K_c x^2_{cy} + K_c x_{cy}x_3 + \frac{1}{2}K_{sc} x^2_3 + \frac{1}{2}K_{ty} x^2_{ty} + K_{ty}x_2 + K_c [(x_{cy} - x_c) - (x_{ty} + x_t) + x_3] \\
+ \frac{1}{2}K_{st} [(x_{cy} - x_c) - (x_{ty} + x_t) + x_3]^2 - Nx_3
\]

and the applied load, together with the component forces, are given thus:

\[
N = K_c x_{cy} + K_{ty} + K_{st} [(x_{cy} - x_c) - (x_{ty} + x_t)] + (K_{sc} + K_{st})x_3 \\
N_t = K_{ty} + K_{st} [(x_{cy} - x_c) - (x_{ty} + x_t) + x_3] \\
N_c = K_c x_{cy} + K_{sc}x_3,
\]

where \(K_{sc}\) is the stiffness of the cable after yielding.
2.3.2. **Case II: Cable yields prior to tube**

Case II refers to the scenario in which the cable yields prior to the tube, which occurs when:

\[
\frac{f_{cy}}{E_c} - \frac{P}{A_c E_c} < \frac{f_{ty}}{E_t} + \frac{P}{A_t E_t},
\]

(13)

The analytical formulations to describe the tensile response of prestressed elements in Case II were derived in a similar manner to Case I; the resulting expressions for the three stages of loading are summarised in Table 1; Figure 4 illustrates the generalised coordinate system used for this scenario. Note that the Stage 1 and Stage 3 expressions are the same as those for Case I, since the tube and cable are both elastic (Stage 1) or plastic (Stage 3), though the end points of Stages 1 and 2 and the expressions within Stage 2 differ.

2.3.3. **Tube and cable yield simultaneously (optimal prestress, \(P_{opt,t}\))**

The optimal prestress level under tensile loading may be defined as the one that causes the tube and the cable to yield simultaneously, since this provides the most extensive elastic range and hence the stiffest response.

The optimal prestress level under tensile applied load, \(P_{opt,t}\), can be derived by equating the expressions for the strain required to yield the pretensioned cable to that of the precompressed tube, as shown in Equation (14):

\[
\left( \frac{f_{cy}}{E_c} - \frac{P_{opt,t}}{A_c E_c} \right) = \frac{f_{ty}}{E_t} + \frac{P_{opt,t}}{A_t E_t},
\]

(14)

rearranging in terms of \(P_{opt,t}\) gives

\[
P_{opt,t} = \frac{A_t A_c}{A_t E_t + A_c E_c} \left( f_{cy} E_t - f_{ty} E_c \right).
\]

(15)
Table 1: Analytical solution for tensile response of prestressed elements in Case II

<table>
<thead>
<tr>
<th>Stage</th>
<th>Onset of loading until cable yielding: $[0 \leq x_1 \leq (x_{cy} - x_c)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = (K_c + K_t)x_1$</td>
</tr>
<tr>
<td></td>
<td>$N_t = K_t(x_1 - x_t)$</td>
</tr>
<tr>
<td></td>
<td>$N_c = K_c(x_1 + x_c)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage</th>
<th>From cable yielding to tube yielding: $[0 \leq x_2 \leq (x_{cy} - x_t) + (x_{ty} + x_c)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = K_c x_{cy} + K_t [-x_t + (x_{cy} - x_c)] + (K_t + K_{sc})x_2$</td>
</tr>
<tr>
<td></td>
<td>$N_t = K_t [-x_t + (x_{cy} - x_c) + x_2]$</td>
</tr>
<tr>
<td></td>
<td>$N_c = K_c x_{cy} + K_{sc}x_3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage</th>
<th>From tube yielding to fracture: $[0 \leq x_3 \leq x_f - (x_{ty} + x_t)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = K_c x_{cy} + K_t x_{ty} + K_{sc} [(x_{ty} + x_t) - (x_{cy} - x_c)] + (K_{sc} + K_{st})x_3$</td>
</tr>
<tr>
<td></td>
<td>$N_t = K_t x_{ty} + K_{st}x_3$</td>
</tr>
<tr>
<td></td>
<td>$N_c = K_c x_{cy} + K_{sc} [(x_{ty} + x_t) - (x_{cy} - x_c) + x_3]$</td>
</tr>
</tbody>
</table>

Note that achieving this optimal prestress level without inducing plastic strains during the prestressing operation itself is not possible for all tube and cable material and geometrical properties. To prevent plastic strains during prestressing, the applied prestress must not exceed the yield load of either the tube or cable, as stated in Equations (16) and (17):

$$P_{opt,t} \leq f_{ty} A_t$$  \hspace{1cm} (16)

and

$$P_{opt,t} \leq f_{cy} A_c.$$  \hspace{1cm} (17)

2.3.4. Sample analytical results and finite element validation

In this sub-section, sample analytical results are presented and comparisons are made with corresponding numerical results generated using the FE software ABAQUS [22]. The material and geometrical properties of the different components used in the analytical and numerical models are given in Table 2. Note that both the tube and cable material stress–strain ($\sigma–\epsilon$) response was modelled as elastic, perfectly–plastic, with no strain hardening.

<table>
<thead>
<tr>
<th>Table 2: Properties of prestressed element in the analytical and numerical model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield strength of tube, $f_{ty}$</td>
</tr>
<tr>
<td>Yield strength of cable, $f_{cy}$</td>
</tr>
<tr>
<td>Young’s modulus of tube, $E_t$</td>
</tr>
<tr>
<td>Young’s modulus of cable, $E_c$</td>
</tr>
<tr>
<td>Cross-sectional area of tube, $A_t$</td>
</tr>
<tr>
<td>Cross-sectional area of cable, $A_c$</td>
</tr>
<tr>
<td>Coefficient of linear expansion of cable, $\alpha_c$</td>
</tr>
<tr>
<td>Initial length of tube and cable, $L_o$</td>
</tr>
</tbody>
</table>

As described in Section 2.2, truss elements were used to model the cable and beam elements were used to model the tube. The two components were tied (all displacement degrees of free-
dom) along the member length using tie constraints. In the first step, prestressing was applied
through thermal loading of the cable which caused the cable to contract, resulting in compression
in the tube and tension in the cable. In the next step, the load was applied incrementally through
displacement control until the ultimate load of the prestressed system was reached.

Two levels of preload were considered, both to compare the analytical and numerical results
as well as to illustrate the fundamental mechanics of the prestressed element under different
prestress levels. Figure 5 shows the results obtained for a system with no prestress.

![Figure 5: Tensile performance of a prestressed element with zero initial prestress.](image)

The numerical and analytical models may be seen to be in very close agreement. For the
chosen section properties, the first reduction in stiffness occurs when the tube yields, while the
ultimate load is reached once both components have yielded. The tube yields prior to the cable
since the chosen cable has a lower Young’s modulus and a higher yield strength than the tube.
The drop in stiffness of the system due to the yielding of the tube is greater than the case when
the cable yields since the axial stiffness ratio $K_t/K_c = 9.2$.

The results for the second preload condition, where the optimal preload $P_{opt,t}$ determined
from Equation 15 was applied, are shown in Figure 6. The material and geometrical properties
are again as given in Table 2, and close agreement between the analytical and numerical results
may be observed. During the prestressing process the cable is subjected to tensile forces and the
tube is subjected to compressive forces of the same magnitude. These preload forces, along with
the corresponding displacements, are plotted in Figure 6 following the sign conventions defined
in Section 2.3.1.

For this prestress level, Figure 6 shows that the initial stiffness of the system is maintained
until the ultimate load is reached, at which point both the cable and the tube yield simultaneously.
As a result of maintaining the initial stiffness, the displacements required to reach the ultimate load are minimized. Increasing prestress levels beyond $P_{\text{opt},t}$ would have the effect of yielding the cable prior to that of the tube and thereby increase the displacements at ultimate load, while a lower prestress also results in higher displacements at ultimate load, but now due to earlier yielding of the tube.

3. Experimental Study

An experimental study was performed in the Structures Laboratory at Imperial College London to verify the analytical and numerical findings on the tensile performance of prestressed cable-in-tube systems. Material testing, prestressing operations and the tensile member tests are described.

3.1. Introduction

A total of 7 cable-in-tube specimens were tested: 4 being non-grouted and 3 being grouted. The key variables of the test programme were the prestress level, presence of grout and the tube to cable cross-sectional area ratio, $A_t/A_c$. The cross-section used for all specimens was a cold-formed $60 \times 60 \times 4$ square hollow section (SHS) of grade S355 steel. The cables were 7 wire prestressing strand of grade Y1860S7 steel. The adopted test specimen labelling system is explained in Figure 7 and a full list of the test specimens and their properties is given in Table 3. Note that three target prestress levels were considered: $P_{\text{nom}}$, $\frac{1}{2}P_{\text{opt},t}$ and $P_{\text{opt},t}$ where $P_{\text{nom}}$ is a nominal prestress of 5 kN to ensure that the cables were not slack during grouting and $P_{\text{opt},t}$ is the optimum prestress described in Section 2.3.3. The achieved (measured) levels of prestress are given in Table 3 and differ slightly from the target values.
3.2. Material tests

3.2.1. Tube

Tensile coupon tests were carried out to determine the basic engineering stress–strain response of the SHS tubes used to fabricate test specimens. Coupons were cut from the flat and corner regions of the cross-section. All flat coupons had a parallel necked region of length 150 mm and width 20 mm. The curved corner coupons were parallel with a nominal length of 320 mm.

An Instron 8802 250 kN hydraulic machine was used to carry out the tensile coupon tests in accordance with EN10002-1 [24]. The machine was driven under strain control at a rate of 0.001%/s up to the 0.2% proof stress and at a rate of 0.002%/s beyond the 0.2% proof stress and up to the ultimate tensile stress, from which point a strain rate of 0.04%/s was applied until fracture. Two linear electrical resistance strain gauges placed on either side of the coupons and used to measure the strain. The applied load and axial displacement and other relevant were recorded at 1-s intervals. The measured weighted average (based on the cross-sectional areas of the flat and corner regions) tensile material properties of the tube are given in Table 4 in which $E$ is the Young’s modulus, $\sigma_{0.2}$ is the 0.2% proof stress, $\sigma_u$ is the ultimate stress, $\epsilon_u$ is the percentage strain at the ultimate tensile stress, $\epsilon_f$ is the percentage strain at fracture over the standard gauge length and $n$ and $n_{0.2,u}$ are the Ramberg–Osgood parameters [25].

3.2.2. Cable

The nonlinear stress–strain curve for the 7-wire strand cable was determined by testing a representative two metre long sample. The cable was passed through end plates at either end and anchored using barrels and wedges. The end plates were bolted onto the testing machine
using M30 socket head cap screws. Figure 3.2.2 shows the end connection for the test specimens, though the same end plate bolting arrangement was also used in testing the cable alone. The test was performed using an Instron 2000 kN machine. The machine was driven under displacement control with a constant displacement rate of 0.5 mm/min until fracture. The diameter of each strand was measured and the cross-sectional area of the 7-wire strand cable determined by summing up the cross-sectional areas of the individual strands. The recorded axial load and displacement were used together with the measured length of sample and its cross-sectional area to determine the experimental stress–strain curve. The measured material properties of the cable are also reported in Table 4.

![Diagram of end plate bolting arrangement](image)

**Figure 8**: End plate bolting arrangement for testing of cable-in-tube systems and cables alone.

<table>
<thead>
<tr>
<th>Component</th>
<th>$E$ (kN/mm$^2$)</th>
<th>$\sigma_{0.2}$ (N/mm$^2$)</th>
<th>$\sigma_u$ (N/mm$^2$)</th>
<th>$n$</th>
<th>$n'_{0.2u}$</th>
<th>$\epsilon_u$ (%)</th>
<th>$\epsilon_f$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube</td>
<td>193</td>
<td>354</td>
<td>408</td>
<td>12.0</td>
<td>2.0</td>
<td>3.2</td>
<td>7.1</td>
</tr>
<tr>
<td>Cable</td>
<td>125</td>
<td>1775</td>
<td>1880</td>
<td>10.2</td>
<td>1.5</td>
<td>4.2</td>
<td>4.2</td>
</tr>
</tbody>
</table>

### 3.2.3. Grout

Some of the test specimens were grouted to reflect common construction practice. The grout provides a bond between the tube and the cable, which reduces reliance on the end anchorages and serves to protect the cables from corrosion. In this study, a Portland cement-based grout, with plasticizer used to enhance flowability, was used. The water to cement ratio was 0.35. Samples were taken from each grout mixture to test strength in accordance with EN 12390-3 [26]. The grout was poured into 100 mm cube moulds and were left to dry for one day. The samples were then removed from the mould and cured in water at room temperature. The cubes were tested at various time intervals using an Automax5 testing machine under load-control at a constant rate of 3 kN/s. Two samples were tested at each time interval and the average cube strength $f_{cu}$ is reported in Table 5. The cable-in-tube specimens were tested 7 days after prestressing and grouting.
Table 5: Strength of grout with time.

<table>
<thead>
<tr>
<th></th>
<th>1 day</th>
<th>3 days</th>
<th>7 days</th>
<th>28 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube strength, $f_{cu}$ (N/mm²)</td>
<td>15</td>
<td>38</td>
<td>53</td>
<td>66</td>
</tr>
</tbody>
</table>

3.3. Preparation of specimens and experimental set-up

The cold-formed $60 \times 60 \times 4$ SHS tubes were cut to the required length of two metres, which is a typical length for a bottom chord element used in the construction of prestressed truss structures. The tubes were welded onto 20 mm thick end plates and stiffeners were added, as shown in Figure 11, to prevent premature failure at the ends. The end plates had been drilled to allow the specimens to be directly bolted on to the testing machine, as illustrated in Figure 3.2.2.

To prevent buckling of the tubes during the prestressing process, specimens with initial pre-stress levels higher than the nominal prestress level, $P_{\text{nom}} (5 \text{ kN})$, had circular connecting collars fitted to the cables at mid-length and at quarter points, as shown in Figure 9. The circular connecting collars were machined to an external diameter equal to the inside dimension of the SHS and had central hole to accommodate the cable. The cable, with connecting collars attached at $L_o/4$, $L_o/2$ and $3L_o/4$ from the member ends, was then inserted into the tube. The buckling length of the test specimens under preload were therefore approximately one quarter of their full length, as discussed in Section 2.2.

Prior to prestressing of the specimens, the fixed end anchor was seated by applying an initial prestress of 5 kN. The prestress was then released to fit a wedge at the stressing end of the specimens. Once both anchors were in place, the specimens were prestressed using a T25 stressing jack. After each specimen was prestressed to the appropriate level, the wedge at the stressing end was hammered to reduce prestress loss due to anchorage slip upon jack removal.

A summary of the geometric properties of the test specimens is shown in Table 6, where $L_o$ is the specimen length, $b$ and $d$ are the outer width and height of the SHS specimens, $t$ is the material thickness, $r_i$ is the internal corner radius and $A_t$ and $A_c$ are the cross-sectional areas of the tube and cable, respectively.

3.4. Prestress application and grouting

During prestressing, a load cell was used to monitor the applied force, while strain gauges, affixed to the four faces of the steel tube, were used to verify the force in the tube and to monitor
Figure 10: Prestressing configuration.

Figure 11: Tension specimen test set-up.

Table 6: Summary of measured properties of cable-in-tube test specimens.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$L_0$ (mm)</th>
<th>$b$ (mm)</th>
<th>$d$ (mm)</th>
<th>$t$ (mm)</th>
<th>$r_i$ (mm)</th>
<th>$A_t$ (mm$^2$)</th>
<th>$A_c$ (mm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-1-NG-5</td>
<td>2000.0</td>
<td>60.1</td>
<td>60.0</td>
<td>3.9</td>
<td>2.0</td>
<td>876</td>
<td>101.7</td>
</tr>
<tr>
<td>T-1-NG-68</td>
<td>2000.0</td>
<td>60.0</td>
<td>60.0</td>
<td>3.9</td>
<td>2.0</td>
<td>875</td>
<td>101.7</td>
</tr>
<tr>
<td>T-1-NG-126</td>
<td>2000.0</td>
<td>59.9</td>
<td>60.1</td>
<td>3.9</td>
<td>2.0</td>
<td>875</td>
<td>101.7</td>
</tr>
<tr>
<td>T-2-NG-108</td>
<td>2000.0</td>
<td>60.0</td>
<td>60.2</td>
<td>3.9</td>
<td>2.0</td>
<td>877</td>
<td>101.7</td>
</tr>
<tr>
<td>T-0-G-0</td>
<td>2000.0</td>
<td>60.0</td>
<td>60.1</td>
<td>3.9</td>
<td>2.0</td>
<td>876</td>
<td>–</td>
</tr>
<tr>
<td>T-1-G-68</td>
<td>2000.0</td>
<td>60.0</td>
<td>60.0</td>
<td>3.9</td>
<td>2.0</td>
<td>875</td>
<td>101.7</td>
</tr>
<tr>
<td>T-1-G-131</td>
<td>2000.0</td>
<td>60.0</td>
<td>59.8</td>
<td>3.9</td>
<td>2.0</td>
<td>874</td>
<td>101.7</td>
</tr>
</tbody>
</table>
any flexure, as shown in Figure 10.

The tension in the cables was used to stabilise the compressed tubular specimens during prestressing, with the collars used to transfer the lateral resistance provided by the cable to the tube, as discussed in Section 2.2. Figure 12 confirms the stabilising effect of the cables during the prestressing process. The strain measurements (on the primary vertical axis) and the prestress level (on the secondary vertical axis) of specimen T-1-G-131, during the prestressing stage, are plotted against the elapsed time in Figure 12 where $\epsilon_a$ and $\epsilon_b$ are the axial and bending strains respectively. Figure 12(a) shows that at an applied prestress level of 96 kN, the bending strain was only 13% of the axial strain whereas the bending strain was 27% of the axial strain when a tube of the same properties was tested under an externally applied compression of the same magnitude. Prestress greater than the target level was initially applied to the specimens to take account of the prestress loss upon jack removal.

Grouting was carried out after prestressing. Specimens that required grouting had holes in the end-plates to allow the grout to flow straight into the tube. Specimens to be grouted were set upright and grout was poured into the specimens at the top end using a funnel. The specimens were tapped, using a metal rod, along the length immediately after the grout was poured to ensure that the grout had filled the tube.

![Figure 12: Prestress level and flexural behaviour of specimens during prestressing stage.](image)

(a) Strain variations for specimen T-1-G-68

(b) Strain gauge locations (SG).

3.5. Experimental results

The full set of experimental results are shown in Figure 13, which gives the tensile load–extension response of the seven specimens. The figure illustrates that the test specimens exhibit the characteristic behaviour predicted by the analytical model discussed in Section 2, whereby the addition of a cable increases the member capacity and increasing the level of prestress up to $P_{opt,t}$ reduces the displacements required to reach the ultimate load (i.e. the initial stiffness is maintained over a greater range. Specimen T-0-G-0 may be considered as a control specimen, against which the beneficial influence of the inclusion of cables and prestressing can be assessed.
As discussed in Section 4.2, the presence of grout has a relatively small influence on tensile structural response, though a less ductile response was typically displayed by these specimens (see Figure 13). For the non-grouted specimens, the cable capacity was lost through the progressive fracture of individual strands followed by fracture of the tube, whereas for the grouted specimens simultaneous fracture of all cable strands was observed, which was followed closely by fracture of the tube.

4. Comparison between experimental, analytical and numerical results

In this section, a detailed description of the experimental results is given and comparisons are made with the analytical and numerical results. The measured geometrical, material and pre-stress levels were implemented in the numerical and analytical models. The two stage Ramberg–Osgood model using the measured $\sigma$–$\epsilon$ response of the cable and tube in Table 4 was adopted in the numerical model, whereas in the analytical model a bi-linear representation of the material was assumed. The latter used the measured 0.2% proof stress, $\sigma_{0.2}$, as $f_y$ and the measured ultimate stress and strain values ($\sigma_u$ and $\epsilon_u$) to determine $E_{st}$ and $E_{sc}$, which are the linear strain hardening moduli of the tube and cable respectively.

4.1. Non-grouted specimens

The experimental load–displacement curves for the non-grouted specimens, together with the corresponding analytical and numerical results, are shown in Figures 14–17. In general, there is close agreement between the theoretical and experimental results, though the sharp change
in slope at the yield point of the tube predicted by the analytical model is less distinct in the experimental results. This is attributed largely to the slightly rounded stress-strain response of the tested cold-formed steel tubes, variation of yield stress between the four faces of the tubes, and small possible deformations or slippage in the test set-up.

Figure 14 shows the results for specimen T-1-NG-5. There is a good correlation between the experimental, numerical and analytical results, but the premature fracture of one strand, in the 7-wire prestressing cable, has resulted in the analytical and numerical models slightly overpredicting the experimental results. From the axial force variation plotted for the tube and cable in Figure 14, it is observed that the addition of a prestressed cable increased the tensile capacity of the system by more than 50%. The experimental results for specimen T-1-NG-68 are shown in Figure 15 and are in very close agreement with the analytical and numerical results. As the figure illustrates, the increased level of prestress in specimen T-1-NG-68 had the effect of increasing the elastic range of the system response which reduced the axial deflection required to reach ultimate load by 30% in comparison with that of specimen T-1-NG-5.

The highest prestress level for the non-grouted elements was applied to specimen T-1-NG-126, the behaviour of which is shown in Figure 16. The figure again demonstrates good agreement between experimental, numerical and analytical results, and shows that the desired simultaneous yield of the tube and cable was almost achieved in the test; compared to specimen T-1-NG-5 the deflection required to reach ultimate load was reduced by approximately 60%.

Figure 14: Experimental results with theoretical and numerical model comparisons for specimen T-1-NG-5.

Specimen T-2-NG-108 included two cables, thus altering the tube to cable cross-sectional area ratio. The experimental load–deflection results, which show a stiffer initial response and
Figure 15: Experimental results with theoretical and numerical model comparisons for specimen T-1-NG-68.

Figure 16: Experimental results with theoretical and numerical model comparisons for specimen T-1-NG-126.
higher load carrying capacity than the single cable specimens, are again well predicted by the analytical and numerical models.

Figure 17: Experimental results with theoretical and numerical model comparisons for specimen T-2-NG-108.

Figure 18: Experimental results with theoretical and numerical model comparisons for specimen T-1-G-68.
Figure 19: Experimental results with theoretical and numerical model comparisons for specimen T-1-G-131.

4.2. Grouted specimens

The load–deflection response of the grouted specimens are shown in Figures 18 and 19. In general, the grouted specimens exhibit similar behaviour to their non-grouted counterparts due to the anticipated limited contribution of the grout in tension. Clearly, a greater contribution would be expected in compression. The small enhancement in capacity of specimen T-1-G-131 over specimen T-1-NG-126 could be attributed to the grout providing restraint against necking of the steel tube and thereby inducing a biaxial tension state, in which the yield strength in the longitudinal direction is improved, as discussed in previous studies on concrete filled steel tubes [27].

5. Conclusions

The tensile performance of prestressed steel cable-in-tube systems has been examined in this paper. The stability of the system under prestress as well as the response of the elements to the subsequent tensile loading were investigated. An existing analytical prediction of the behaviour under preload was verified both numerically and experimentally. For the latter, connecting collars between the cable and the tube located at intervals along the member length were shown to be able to transfer the restraining forces provided by the tensioned cable to the tube and restrict overall buckling. Hence provided the cable is in continuous contact with the tube, or a sufficient number of connecting collars are utilised, buckling under preload is effectively prevented.

An analytical model to predict the tensile load–deflection response of the prestressed elements was then established, with the key controlling parameters being the yield loads of the tube and cable, the pre- and post-yield stiffnesses of the tube and cable, and the level of preload. An
optimum prestress level, corresponding to simultaneous yield of the tube and cable was also defined; at this prestress level, the axial deformation corresponding to the yielding of the system was minimised. The analytical predictions were verified by comparison with experimental and numerical results. The full load–deflection curves determined by means of the three approaches (experimental, numerical and analytical) were shown to be in close agreement, and the benefits of the inclusion of the cables and their prestressing, in terms of both enhanced strength and stiffness, were demonstrated.

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References


